Homework 1

① $f(c)\,dc = 0$

$$f(c) = 4\pi c^2 \left(\frac{m}{2\pi RT}\right)^{\frac{3}{2}} e^{-mc^2/2RT}$$

$$f(c)\,dc = e^{-mc^2/2RT}\left[8\pi c\left(\frac{m}{2\pi RT}\right)^{3/2} + \left(-\frac{2mc}{2RT}\right)4\pi c^2\left(\frac{m}{2\pi RT}\right)^{\frac{3}{2}}\right]$$

$$= e^{-mc^2/2RT}\left[\left(\frac{m}{2\pi RT}\right)^{\frac{3}{2}} 4\pi c\left[2 - \frac{\cancel{2}\,mc^2}{\cancel{2}\,RT}\right]\right] = 0$$

$$2 - \frac{Mc^2}{RT} = 0, \quad \frac{Mc^2}{RT} = 2, \quad c^2 = \frac{2RT}{M}$$

$$c_{mp} = (2RT/M)^{1/2}$$

② $\langle c^2\rangle = \int c^2 f(c)\,dc$ $\qquad f(c) = 4\pi c^2\left(\frac{M}{2\pi RT}\right)^{3/2} e^{-Mc^2/2RT}$

$$\langle c^2\rangle = \int_0^\infty c^2\left[4\pi c^2\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} e^{-Mc^2/2RT}\right]dc = \int_0^\infty 4\pi c^4\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} e^{-Mc^2/2RT}\,dc$$

$n = 2$, $a = M/2RT$, $x = c$, $\int_0^\infty x^{2n}\exp(-ax^2)\,dx = (2n)!\,\pi^{\frac{1}{2}}/(2^{2n+1} n!\,a^{n+\frac{1}{2}})$

$$\Rightarrow \langle c^2\rangle = 4\pi\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}}\frac{4!\,\pi^{\frac{1}{2}}(2RT)^{5/2}}{2^5 \times 2! \times M^{5/2}}$$

$$= \cancel{4}\pi\left(\frac{M}{2\pi RT}\right)^{3/2}\frac{\cancel{4}\times 3\times\cancel{2}\times 1\times\pi^{\frac{1}{2}}(2RT)^{5/2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}\times 2\times 1\times M^{5/2}}$$

$$= 3\pi^{\frac{3}{2}}\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}}\left(\frac{2RT}{M}\right)^{\frac{5}{2}} = \boxed{\frac{3RT}{M}}$$

(3) 1 gallon of water = $3785.4 cm^3$ water

water density in 25°c = 0.997 gr/cm^3 $[d = \frac{m}{v}]$

$\Rightarrow$ m = 0.997 × 3785.4 = 3774.04 gr

Molar mass of water = 18 gr/mol $\longrightarrow$ 1 mol (w) = 18 gr

1 mol = 6.022×10^{23} molecules

$\Rightarrow \frac{3774.04 \text{ gr}}{18 \text{ gr/mol}} = 209.669$ mol

total # of water molecules in a gallon = $209669 \text{ mol} \times 6.022 \times 10^{23} \frac{\text{molecules}}{\text{mol}}$

$= 1.263 \times 10^{23}$ molecules

4.

For CO_2, $a = 3.60$ atm L^2 mol^{-2} and $b = 0.0427$ L mol^{-1}. Rearrange the van der Waals equation, $\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$ to give

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$= \frac{(2.500 \text{ mol})(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})(450 \text{ K})}{1.000 \text{ L} - (2.500 \text{ mol})(0.0427 \text{ L mol}^{-1})} - \frac{(3.60 \text{ atm L}^2 \text{ mol}^{-2})(2.500 \text{ mol})^2}{(1.000 \text{ L})^2}$$

$= 80.9$ atm

If CO_2 behaved ideally,

$$P = \frac{nRT}{V} = \frac{(2.500 \text{ mol})(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})(450 \text{ K})}{1.000 \text{ L}} = 92.3 \text{ atm}$$

The pressure calculated using the van der Waals equation of state is smaller than that calculated using the ideal gas equation. Thus, under these conditions there are net attractive forces between molecules.